

Axion-Higgs interplay in the two Higgs-doublet model

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Abstract

The DFSZ model is a natural extension of the two-Higgs doublet model containing an additional singlet, endowed with a Peccei-Quinn symmetry, and leading to a physically acceptable axion. In this paper we re-examine this model in the light of some new developments. For generic couplings the model reproduces the minimal Standard Model showing only tiny deviations (extreme decoupling scenario) and all additional degrees of freedom (with the exception of the axion) are very heavy. Recently it has been remarked that the limit where the coupling between the singlet and the two doublets becomes very small is technically natural. Combining this limit with the requirement of exact or approximate custodial symmetry we may obtain an additional 0^+ Higgs at the weak scale, accompanied by relatively light charged and neutral pseudoscalars. The mass spectrum would then resemble that of a generic two Higgs-doublet model, with naturally adjustable masses in spite of the large scale that the axion introduces. However the couplings are non-generic in this model. We use the recent constraints derived from the Higgs-WW coupling together with oblique corrections to constrain the model as much as possible. As an additional result, we work out the non-linear parametrization of the DFSZ model in the generic case where all scalars except the lightest Higgs and the axion have masses at or beyond the TeV scale.

I. INTRODUCTION

An invisible axion [1–3] constitutes to this date a very firm candidate to provide all or part of the dark matter component of the cosmological budget. There are several extensions of the Minimal Standard Model (MSM) providing a particle with the characteristics and couplings of the axion [4, 5]. In our view a particularly interesting possibility is the model suggested by Zhitnitsky and Dine, Fischler and Srednicki (DFSZ) more than 30 years ago that consists in a fairly simple extension of the popular two Higgs-doublet model (2HDM). As a matter of fact it could probably be argued that a good motivation to consider the 2HDM is that it allows for the inclusion of a (nearly) invisible axion [6–8]. Of course there are other reasons why the 2HDM should be considered as a possible extension of the MSM. Apart from purely aesthetic reasons, it is easy to reconcile such models with existing constraints. They may give rise to a rich phenomenology, including possibly (but not necessarily) flavour changing neutral currents at some level, custodial symmetry breaking terms or even new sources of CP violation [9, 10].

Following the discovery of a Higgs-like particle with $m_h \sim 125$ GeV there have been a number of works considering the implications of such a finding on a generic 2HDM, together with the constraints arising from the lack of detection of new scalars and from the electroweak precision observables [11]. Depending on the way that the two doublets couple to fermions, they are classified as type I, II or III (see e.g. [9] for details), with different implications on the flavour sector. Consideration of all the different types of 2HDM plus all the rich phenomenology that can be encoded in the Higgs potential leads to a wide variety of possibilities with different experimental implications, even after applying all the known phenomenological low-energy requirements.

Requiring a Peccei-Quinn (PQ) symmetry leading to an axion does however severely restrict the possibilities, and this is in our view an important asset of the DFSZ model. This turns out to be particularly the case when one includes all the recent experimental information concerning the 125 GeV scalar state and its couplings. Exploring this model, taking into account all these constraints is the motivation for the present work.

The structure of this paper is as follows. In section II we discuss the possible global symmetries of the DFSZ model, namely $U(1)_{PQ}$ (always present), and $SU(2)_L \times SU(2)_R$ (the $SU(2)_R$ subgroup may or may not be present). Symmetries are best discussed by using

a matrix formalism that we review and extend. Section III is devoted to the determination of the spectrum of the theory. We present up to four generic cases that range from the extreme decoupling, where the model –apart from the presence of the axion– is indistinguishable from the MSM at low energies, to one where there are extra light Higgses below or around the TeV scale. This last case necessarily requires some couplings in the potential to be very small; a possibility that is nevertheless natural in a technical sense and therefore should be contemplated as a viable theoretical hypothesis. We discuss in detail the situation where custodial symmetry is exact or approximately valid in this model because the combination of this symmetry and naturally small couplings allows us to keep the additional scalars ‘naturally light’ if we so wish with only one exception, meaning that the ‘contamination’ from the large scale present in the theory is under control.

However, while additional scalars may exist at or just above the weak scale in this model, they can also be made heavy, with masses in the multi-TeV region or beyond. In section IV we discuss the resulting non-linear effective theory emerging in this generic situation.

Next in section V we analyze the impact of the model on the (light) Higgs effective couplings to gauge bosons and the constraints that can be derived from the recent LHC data. In section VI we compare the potential of the DFSZ model with the most general potential in the 2HDM. We find out which terms of the general potential are forbidden by the PQ symmetry and which ones are recovered when it is spontaneously broken by the VEV of the scalar fields. Finally in section VII the restrictions that the electroweak precision parameters, particularly $\Delta\rho$, impose on the model are discussed. These restrictions are relevant only in the case where all or part of the additional spectrum of scalars is light as we find that they are automatically satisfied otherwise.

We would like to emphasize that even after imposing the constraints derived from the PQ symmetry the model still contains enough degrees of freedom to reproduce the mass spectrum of a generic 2HDM, so there is no predictivity at the level of the spectrum. However, this nearly exhausts all freedom available, particularly if exact or approximate custodial symmetry is imposed. Then the scalar couplings are largely fixed and in this sense the model is far more predictive than a generic 2HDM —and in addition it contains the axion, which is its *raison d’être*.

II. MODEL AND SYMMETRIES

The DFSZ model contains two Higgs doublets and one complex scalar singlet, namely

$$\phi_1 = \begin{pmatrix} \alpha_+ \\ \alpha_0 \end{pmatrix}; \quad \phi_2 = \begin{pmatrix} \beta_+ \\ \beta_0 \end{pmatrix}; \quad \phi, \quad (1)$$

with vacuum expectation values (VEVs) $\langle \alpha_0 \rangle = v_1$, $\langle \beta_0 \rangle = v_2$, $\langle \phi \rangle = v_\phi$ and $\langle \alpha_+ \rangle = \langle \beta_+ \rangle = 0$. Moreover, we define the usual electroweak vacuum expectation value $v = 246$ GeV as $v^2 = (v_1^2 + v_2^2)/2$ and $\tan \beta = v_2/v_1$. The implementation of the PQ symmetry is only possible for type II models, where the Yukawa terms are

$$\mathcal{L}_Y = G_1 \bar{q}_L \tilde{\phi}_1 u_R + G_2 \bar{q}_L \phi_2 d_R + h.c., \quad (2)$$

with $\tilde{\phi}_i = i\tau_2 \phi_i^*$. The PQ transformation acts on the scalars as

$$\phi_1 \rightarrow e^{iX_1\theta} \phi_1, \quad \phi_2 \rightarrow e^{iX_2\theta} \phi_2, \quad \phi \rightarrow e^{iX_\phi\theta} \phi \quad (3)$$

and on the fermions as

$$q_L \rightarrow q_L, \quad l_L \rightarrow l_L, \quad u_R \rightarrow e^{iX_u\theta} u_R, \quad d_R \rightarrow e^{iX_d\theta} d_R, \quad e_R \rightarrow e^{iX_e\theta} e_R. \quad (4)$$

For the Yukawa terms to be PQ-invariant we need

$$X_u = X_1, \quad X_d = -X_2, \quad X_e = -X_2. \quad (5)$$

Let us now turn to the potential involving the two doublets and the new complex singlet.

The most general potential compatible with PQ symmetry is

$$\begin{aligned} V(\phi, \phi_1, \phi_2) = & \lambda_\phi (\phi^* \phi - V_\phi^2)^2 + \lambda_1 (\phi_1^\dagger \phi_1 - V_1^2)^2 + \lambda_2 (\phi_2^\dagger \phi_2 - V_2^2)^2 \\ & + \lambda_3 (\phi_1^\dagger \phi_1 - V_1^2 + \phi_2^\dagger \phi_2 - V_2^2)^2 + \lambda_4 \left[(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right] \\ & + (a\phi_1^\dagger \phi_1 + b\phi_2^\dagger \phi_2)\phi^* \phi - c(\phi_1^\dagger \phi_2 \phi^2 + \phi_2^\dagger \phi_1 \phi^{*2}) \end{aligned} \quad (6)$$

The c term imposes the condition $-X_1 + X_2 + 2X_\phi = 0$. If we impose that the PQ current does not couple to the Goldstone boson that is eaten by the Z , we also get $X_1 \cos^2 \beta + X_2 \sin^2 \beta = 0$. If furthermore we choose¹ $X_\phi = -1/2$ the PQ charges of the doublets are

$$X_1 = -\sin^2 \beta, \quad X_2 = \cos^2 \beta. \quad (7)$$

¹ There is arbitrariness in this choice. This election conforms to the conventions existing in the literature.

Global symmetries are not very evident in the way fields are introduced above. To remedy this let us define the matrices [12]

$$\Phi_{12} = (\tilde{\phi}_1 \ \phi_2) = \begin{pmatrix} \alpha_0^* & \beta_+ \\ -\alpha_- & \beta_0 \end{pmatrix}, \quad \Phi_{21} = (\tilde{\phi}_2 \ \phi_1) = \begin{pmatrix} \beta_0^* & \alpha_+ \\ -\beta_- & \alpha_0 \end{pmatrix} = \tau_2 \Phi_{12}^* \tau_2 \quad (8)$$

and

$$I = \Phi_{12}^\dagger \Phi_{12} = \begin{pmatrix} \phi_1^\dagger \phi_1 & \tilde{\phi}_1^\dagger \phi_2 \\ -\phi_1^\dagger \tilde{\phi}_2 & \phi_2^\dagger \phi_2 \end{pmatrix}, \quad J = \Phi_{12}^\dagger \Phi_{21} = \phi_2^\dagger \phi_1 \mathbf{I}. \quad (9)$$

Defining also the constant matrix $W = (V_1^2 + V_2^2)\mathbf{I}/2 + (V_1^2 - V_2^2)\tau_3/2$, we can write the potential as

$$\begin{aligned} V(\phi, I, J) = & \lambda_\phi (\phi^* \phi - V_\phi^2)^2 + \frac{\lambda_1}{4} \{ \text{Tr} [(I - W)(1 + \tau_3)] \}^2 \\ & + \frac{\lambda_2}{4} \{ \text{Tr} [(I - W)(1 - \tau_3)] \}^2 + \lambda_3 [\text{Tr}(I - W)]^2 \\ & + \frac{\lambda_4}{4} \text{Tr} [I^2 - (I\tau_3)^2] + \frac{1}{2} \text{Tr} [(a + b)I + (a - b)I\tau_3] \phi^* \phi \\ & - \frac{c}{2} \text{Tr}(J\phi^2 + J^\dagger \phi^{*2}). \end{aligned} \quad (10)$$

A $SU(2)_L \times SU(2)_R$ global transformation acts on our fields as

$$\Phi_{ij} \rightarrow L\Phi_{ij}R^\dagger, \quad I \rightarrow RIR^\dagger, \quad J \rightarrow J. \quad (11)$$

We now we are in a better position to discuss the global symmetries of the potential. The behavior of the different parameters under $SU(2)_R$ is shown in Table I. See also [13].

Finally, let us establish the action of the PQ symmetry previously discussed in this parametrization. Under the PQ transformation:

$$\Phi_{12} \rightarrow \Phi_{12} e^{iX\theta}, \quad \phi \rightarrow e^{iX_\phi\theta} \phi \quad (12)$$

with

$$X = \frac{X_2 - X_1}{2} \mathbf{I} - \frac{X_2 + X_1}{2} \tau_3, \quad X_\phi = \frac{X_2 - X_1}{2} \quad (13)$$

Using the values for $X_{1,2}$ given in Eq. (7)

$$X = \begin{pmatrix} \sin^2 \beta & 0 \\ 0 & \cos^2 \beta \end{pmatrix}, \quad X_\phi = -\frac{1}{2}. \quad (14)$$

Parameter	Custodial limit
$\lambda_1, \lambda_2, \lambda_4$	$\lambda_1 = \lambda_2 = \lambda$ and $\lambda_4 = 2\lambda$
λ_3	λ_3
λ_ϕ	λ_ϕ
V_1^2, V_2^2	$V_1^2 = V_2^2 = V^2$
V_ϕ	V_ϕ
a, b	$a = b$
c	c

TABLE I: In total, there are 11 parameters: 7 are custodially preserving and 4 are custodially breaking. See the text for our usage of the expression ‘custodial symmetry’ in the context of a 2HDM.

III. MASSES AND MIXINGS

We have two doublets and a singlet, so a total of $4 + 4 + 2 = 10$ spin-zero particles. Three particles are eaten by the W^\pm and Z and 7 scalar fields are left in the spectrum; two charged Higgs, two 0^- states and three neutral 0^+ states. Our field definitions will be worked out in full detail in section IV. Here we want only to derive the spectrum. For the charged Higgs mass we have at tree level ²

$$m_{H^\pm}^2 = 8 \left(\lambda_4 v^2 + \frac{c v_\phi^2}{s_{2\beta}} \right). \quad (15)$$

The quantity v_ϕ is proportional to the axion decay constant. Its value is known to be very large (at least 10^7 GeV and probably substantially larger $\sim 10^9$ GeV if all astrophysical constraints are taken into account, see [14] for several experimental and cosmological bounds). It does definitely make sense to organize the calculations as an expansion in v/v_ϕ .

In the 0^- sector there are two degrees of freedom that mix with each other with a mass matrix which has a vanishing eigenvalue. The eigenstate with zero mass is the axion and A_0 is the pseudoscalar Higgs with mass

$$m_{A_0}^2 = 8c \left(\frac{v_\phi^2}{s_{2\beta}} + v^2 s_{2\beta} \right). \quad (16)$$

² Here and in the following we introduce the short-hand notation $s_{m\beta}^n \equiv \sin^n(m\beta)$ and $c_{m\beta}^n \equiv \cos^n(m\beta)$.

Eq. (16) implies $c \geq 0$. For $c = 0$, the mass matrix in the 0^- sector has a second zero, i.e. in practice the A_0 field behaves as another axion.

In the 0^+ sector, there are three neutral particles that mix with each other. With h_i we denote the corresponding 0^+ mass eigenstates. The mass matrix is given in Appendix B. In the limit of large v_ϕ , the mass matrix in the 0^+ sector can be easily diagonalized [7] and presents one eigenvalue nominally of order v^2 and two of order v_ϕ^2 . Up to $\mathcal{O}(v^2/v_\phi^2)$, these masses are

$$m_{h_1}^2 = 32v^2 (\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_3) - 16v^2 \frac{(ac_\beta^2 + bs_\beta^2 - cs_{2\beta})^2}{\lambda_\phi}, \quad (17)$$

$$m_{h_2}^2 = \frac{8c}{s_{2\beta}} v_\phi^2 + 8v^2 s_{2\beta}^2 (\lambda_1 + \lambda_2) - 4v^2 \frac{[(a-b)s_{2\beta} + 2cc_{2\beta}]^2}{\lambda_\phi - 2c/s_{2\beta}}, \quad (18)$$

$$m_{h_3}^2 = 4\lambda_\phi v_\phi^2 + 4v^2 \frac{[(a-b)s_{2\beta} + 2cc_{2\beta}]^2}{\lambda_\phi - 2c/s_{2\beta}} + 16v^2 \frac{(ac_\beta^2 + bs_\beta^2 - cs_{2\beta})^2}{\lambda_\phi}. \quad (19)$$

The field h_1 is naturally identified with the scalar boson of mass 125 GeV observed at the LHC.

It is worth it to stress that there are several situations where the above formulae are non-applicable, since the nominal expansion in powers of v/v_ϕ may fail. This may be the case where the coupling constants a, b, c connecting the singlet to the usual 2HDM are very small, of order say v/v_ϕ or v^2/v_ϕ^2 . One should also pay attention to the case $\lambda_\phi \rightarrow 0$ (we have termed this latter case as the ‘quasi-free singlet limit’). Leaving this last case aside, we have found that the above expressions for m_{h_i} apply in the following situations:

Case 1: The couplings a, b and c are generically of $\mathcal{O}(1)$,

Case 2: a, b or c are of $\mathcal{O}(v/v_\phi)$.

Case 3: a, b or c are of $\mathcal{O}(v^2/v_\phi^2)$ but $c \gg \lambda_i v^2/v_\phi^2$.

If $c \ll \lambda_i v^2/v_\phi^2$ the 0^- state is lighter than the lightest 0^+ Higgs and this case is therefore already phenomenologically unacceptable. The only other case that deserves a separate discussion is

Case 4: Same as in case 3 but $c \sim \lambda_i v^2/v_\phi^2$

In this case, the masses in the 0^+ sector read, up to $\mathcal{O}(v^2/v_\phi^2)$, as

$$m_{h_1, h_2}^2 = 8v^2 \left(K \mp \sqrt{K^2 - L} \right) \text{ and } m_{h_3}^2 = 4\lambda_\phi v_\phi^2, \quad (20)$$

where

$$\begin{aligned} K &= 2 \left(\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + \lambda_3 \right) + \frac{c v_\phi^2}{2 v^2 s_{2\beta}}, \\ L &= 4 \left[(\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3) s_{2\beta}^2 + \frac{c v_\phi^2}{v^2 s_{2\beta}} (\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_3) \right]. \end{aligned} \quad (21)$$

Recall that here we assume c to be of $\mathcal{O}(v^2/v_\phi^2)$. Note that

$$m_{h_1}^2 + m_{h_2}^2 = 32 v^2 \left(\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + \lambda_3 + \frac{c v_\phi^2}{4 v^2 s_{2\beta}} \right). \quad (22)$$

In the ‘quasi-free singlet’ limit, when $\lambda_\phi \rightarrow 0$ or more generically $\lambda_\phi \ll a, b, c$ it is impossible to sustain the hierarchy $v \ll v_\phi$, so again this case is phenomenologically uninteresting (see Appendix C for details).

We note that once we set $\tan \beta$ to a fixed value, the lightest Higgs to 125 GeV and v_ϕ to some large value compatible with the experimental bounds, the mass spectrum in Eq. (15), (16) and (17)-(19) grossly depends on the parameters: c , λ_4 and λ_ϕ , the latter only affecting the third 0^+ state that is anyway very heavy and definitely out of reach of LHC experiments; therefore the spectrum depends on only two parameters. If case 4 is applicable, the situation is slightly different and an additional combination of parameters dictates the mass of the second (lightish) 0^+ state. This can be seen in the sum rule of Eq. (22) after requiring that $m_{h_1} = 125$ GeV. Actually this sum rule is also obeyed in cases 1, 2 and 3, but the r.h.s is dominated then by the contribution from the parameter c alone.

A. Heavy and light states

Here we want to discuss the spectrum of the theory according to the different scenarios that we have alluded to in the previous discussion. Let us remember that it is always possible to identify one of the Higgses as the scalar boson found at LHC, namely h_1 .

Case 1. all Higgses except h_1 acquire a mass of order v_ϕ . This includes the charged and 0^- scalars, too. We term this situation ‘extreme decoupling’. The only light states are h_1 , the gauge sector and the massless axion. This is the original DFSZ scenario [4]

Case 2. This situation is similar to case 1 but now the typical scale of masses of h_2 , H_\pm and A_0 is $\sqrt{v v_\phi}$. This range is beyond the LHC reach but it could perhaps

be explored with an accelerator in the 100 TeV region, a possibility being currently debated. Again the only light particles are h_1 , the axion and the gauge sector. This possibility is natural in a technical sense as discussed in [7] as an approximate extra symmetry would protect the hierarchy.

Cases 3 and 4 are phenomenologically more interesting. Here we can at last have new states at the weak scale. In the 0^+ sector, h_3 is definitely very heavy but $m_{h_1}^2$ and $m_{h_2}^2$ are proportional to v^2 once we assume that $c \sim v^2/v_\phi^2$. Depending on the relative size of λ_i and cv_ϕ^2/v^2 one would have to use Eq. (17) or (20). Because in case 3 one assumes that cv_ϕ^2/v^2 is much larger than λ_i , h_1 would still be the lightest Higgs and m_{h_2} could easily be in the TeV region. When examining case 4 it would be convenient to use the sum rule (22).

We note that in case 4 the hierarchy between the different couplings is quite marked: typically to be realized one needs $c \sim 10^{-10}\lambda_i$, where λ_i is a generic coupling of the potential. It is the smallness of this number what results in the presence of light states at the weak scale. For a discussion on the ‘naturalness’ of this possibility see [7].

B. Custodially symmetric potential

While in the usual one doublet model, if we neglect the Yukawa couplings and set the $U_Y(1)$ interactions to zero, custodial symmetry is automatic, the latter is somewhat unnatural in 2HDM as one can write a fairly general potential. These terms are generically not invariant under global transformations $SU(2)_L \times SU(2)_R$ and therefore in the general case after the breaking there is no custodial symmetry to speak of. Let us consider now the case where a global symmetry $SU(2)_L \times SU(2)_R$ is nevertheless present as there are probably good reasons to consider this limit. We may refer somewhat improperly to this situation as to being ‘custodially symmetric’ although after the breaking custodial symmetry proper may or may not be present. If $SU(2)_L \times SU(2)_R$ is to be a symmetry, the parameters of the potential have to be set according to the custodial relations in Table I. Now, there are two possibilities to spontaneously break $SU(2)_L \times SU(2)_R$ and to give mass to the gauge bosons.

1. $SU(2) \times SU(2) \rightarrow U(1)$

If the VEVs of the two Higgs fields are different ($\tan \beta \neq 1$), the custodial symmetry is spontaneously broken to $U(1)$. In this case, one can use the minimization equations of Appendix A to eliminate V , V_ϕ and c of Eq. (10). c turns out to be of order $(v/v_\phi)^2$. In this case there are two extra Goldstone bosons: the charged Higgs is massless

$$m_{H_\pm}^2 = 0. \quad (23)$$

Furthermore, the A_0 is light:

$$m_{A_0}^2 = 16\lambda v^2 \left(1 + \frac{v^2}{v_\phi^2} s_{2\beta}^2 \right) \quad (24)$$

This situation is clearly phenomenologically excluded.

2. $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$

In this case, the VEVs of the Higgs doublets are equal, so $\tan \beta = 1$. The masses are

$$m_{H_\pm}^2 = 8(2\lambda v^2 + cv_\phi^2), \quad m_{A_0}^2 = 8c(v^2 + v_\phi^2) \quad \text{and} \quad m_{h_2}^2 = m_{H_\pm}^2. \quad (25)$$

These three states are parametrically heavy, but they may be light in cases 3 and 4.

The rest of the 0^+ mass matrix is 2×2 and has eigenvalues (up to second order in v/v_ϕ)

$$m_{h_1}^2 = 16v^2 \left[\lambda + 2\lambda_3 - \frac{(a-c)^2}{\lambda_\phi} \right] \quad \text{and} \quad m_{h_3}^2 = 4 \left[\lambda_\phi v_\phi^2 + 4v^2 \frac{(a-c)^2}{\lambda_\phi} \right]. \quad (26)$$

It is interesting to explore in this relatively simple case what sort of masses can be obtained by varying the values of the couplings in the potential (λ , λ_3 and c). We are basically interested in the possibility of obtaining a lightish spectrum (case 4 previously discussed) and accordingly we assume that the natural scale of c is $\sim v^2/v_\phi^2$. We have to require the stability of the potential discussed in Appendix D as well as $m_{h_1} = 125$ GeV. The allowed region is shown in Fig. 1. Since we are in a custodially symmetric case there are no further restrictions to be obtained from $\Delta\rho$.

C. Understanding hierarchies

As is well known in the MSM the Higgs mass has potentially large corrections if the MSM is understood as an effective theory and one assumes that a larger scale must show up in the

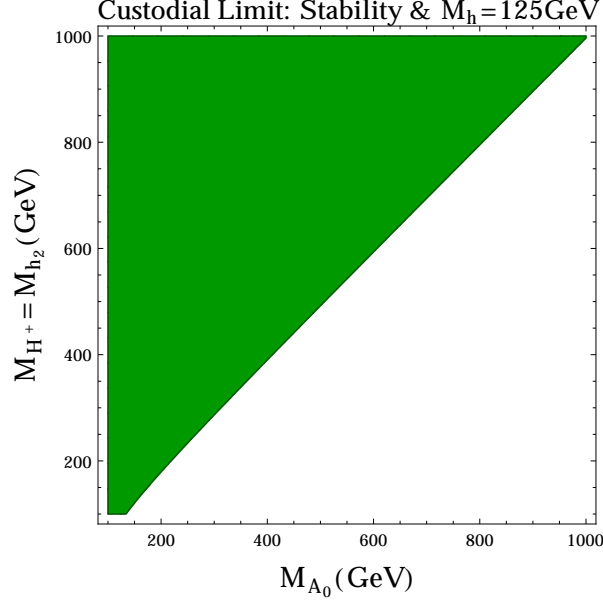


FIG. 1: Dark/green: allowed region in the custodial limit after requiring vacuum stability (see e.g. Appendix D). Each point in this region corresponds to a valid set of parameters in the DFSZ potential. Note that c is assumed to be of order v^2/v_ϕ^2 and cv_ϕ^2/v^2 has to be $\sim \lambda_i$ (case 4 discussed in the text).

theory at some moment. This is the case, for instance, if neutrino masses are included via the see-saw mechanism, to name just a possibility. In this case to keep the 125 GeV Higgs light one must do some amount of fine tuning.

In the DFSZ model such a large scale is indeed present from the outset and consequently one has to envisage the possibility that the mass formulae previously derived may be subject to large corrections due to the fact that v_ϕ leaks in the low energy scalar spectrum. Let us discuss the relevance of the hierarchy problem in the different cases discussed in this section.

In case 1 all masses in the scalar sector but the physical Higgs are heavy, of order v_ϕ , and due to the fact that the couplings λ_i in the potential are generic (and also the couplings a, b, c connecting the two doublets to the singlet) the hierarchy may affect the light Higgs quite severely and fine tuning of the λ_i will be called for. However this fine tuning is not essentially different from the one commonly accepted to be necessary in the MSM to keep the Higgs light if a new scale is somehow present.

In cases 3 and 4 the amount of additional fine tuning needed is none or very little. In these scenarios (particularly in case 4) the scalar spectrum is light, in the TeV region, and the only

heavy degree of freedom is contained in the modulus of the singlet. After diagonalization this results in a very heavy 0^+ state (h_3), with a mass of order v_ϕ . However inspection of the potential reveals that this degree of freedom couples to the physical sector with a strength v^2/v_ϕ^2 and therefore may change the tree-level masses by a contribution of order v —perfectly acceptable. In this sense the ‘natural’ scenario proposed in [7] does not apparently lead to a severe hierarchy problem in spite of the large scale involved.

Case 2 is particularly interesting. In this case the intermediate masses are of order $\sqrt{vv_\phi}$, i.e. ~ 100 TeV. There is still a very heavy mass eigenstate (h_3) but again is nearly decoupled from the lightest Higgs as in cases 3 and 4. On the contrary, the states with masses $\sim \sqrt{vv_\phi}$ do couple to the light Higgs with strength $\sim \lambda_i$ and thus require —thanks to the loop suppression factor— only a very moderate amount of fine tuning as compared to case 1.

It is specially relevant in the context of the hierarchy problem to consider the custodial case discussed in the previous section. In the custodial limit the A_0 mass is protected as it is proportional to the extended symmetry breaking parameter c . In addition $m_{h_2} = m_{H^\pm}$. Should one wish to keep a control on radiative corrections, doing the fine tuning necessary to keep h_1 and h_2 light should suffice and in fact the contamination from the heavy h_3 is limited as said above. Of course, to satisfy the present data we have to worry only about h_1 .

IV. NON-LINEAR EFFECTIVE LAGRANGIAN

We have seen in the previous section that the spectrum of scalars resulting from the potential of the DFSZ model is generically heavy. It is somewhat difficult to have all the scalar masses at the weak scale, although the additional scalars can be made to have masses in weak scale region in case 4. The only exceptions are the three Goldstone bosons, the h_1 Higgs and the axion. It is therefore somehow natural to use a non-linear realization to describe the low energy sector formed by gauge bosons (and their associated Goldstone bosons), the lightest Higgs 0^+ state h_1 , and the axion. Deriving this effective action is one of the motivations of this work.

To construct the effective action we have to identify the proper variables and in order to do so we will follow the technique described in [12]. In that paper the case of a generic 2HDM where all scalar fields were very massive was considered. Now we have to modify the

method to allow for a light state (the h_1) and also to include the axion degree of freedom.

We decompose the matrix-valued Φ_{12} field introduced in Section II in the following form

$$\Phi_{12} = \mathcal{U} \mathcal{M}_{12}. \quad (27)$$

\mathcal{U} is a 2×2 matrix that contains the three Goldstone bosons associated to the breaking of $SU(2)_L$ (or more precisely of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$). We denote by G^i these Goldstone bosons

$$\mathcal{U} = \exp \left(i \frac{\vec{G} \cdot \vec{\tau}}{v} \right). \quad (28)$$

Note that the matrices I and J of Eq. (9) entering the DFSZ potential are actually independent of \mathcal{U} . This is immediate to see in the case of I while for J one has to use the property $\tau_2 \mathcal{U}^* = \mathcal{U} \tau_2$ valid for $SU(2)$ matrices. The effective potential then does depend only on the degrees of freedom contained in \mathcal{M}_{12} whereas the Goldstone bosons drop from the potential, since, under a global $SU(2)_L \times SU(2)_R$ rotation, Φ_{12} and \mathcal{U} transform as

$$\Phi_{12} \rightarrow L \Phi_{12} R^\dagger \quad \mathcal{U} \rightarrow L \mathcal{U} R^\dagger \Rightarrow \mathcal{M}_{12} \rightarrow R \mathcal{M}_{12} R^\dagger. \quad (29)$$

Obviously the same applies to the locally gauged subgroup.

Let us now discuss the potential and \mathcal{M}_{12} further. Inspection of the potential shows that because of the term proportional to c the phase of the singlet field ϕ does not drop automatically from the potential and thus it cannot be immediately identified with the axion. In other words, the phase of the ϕ field mixes with the usual 0^- scalar from the 2HDM. To deal with this let us find a suitable phase both in \mathcal{M}_{12} and in ϕ that drops from the effective potential – this will single out the massless state to be identified with the axion.

We write $\mathcal{M}_{12} = M_{12} U_a$, where U_a is a unitary matrix containing the axion. An immediate choice is to take the generator of U_a to be the identity, which obviously can remove the phase of the singlet in the term in the effective potential proportional to c while leaving the other terms manifestly invariant. This does not exhaust all freedom however as we can include in the exponent of U_a a term proportional to τ_3 . It can be seen immediately that this would again drop from all the terms in the effective potential, including the one proportional to c when taking into account that ϕ is a singlet under the action of τ_3 that of course is nothing but the hypercharge generator. We will use the remaining freedom just discussed to properly normalize the axion and A_0 fields in the kinetic terms to which we now turn.

The gauge invariant kinetic term will be

$$\mathcal{L}_{\text{kin}} = \frac{1}{2}(\partial_\mu \phi)^* \partial^\mu \phi + \frac{1}{4} \text{Tr} \left[(D_\mu \Phi_{12}^\dagger) D^\mu \Phi_{12} \right], \quad (30)$$

where the covariant derivative is defined by

$$D_\mu \Phi_{12} = \partial_\mu \Phi_{12} - i \frac{g}{2} \vec{W}_\mu \cdot \vec{\tau} \Phi_{12} + i \frac{g'}{2} B_\mu \Phi_{12} \tau_3. \quad (31)$$

By defining $U_a = \exp \left(2ia_\phi X / \sqrt{v_\phi^2 + v^2 s_{2\beta}^2} \right)$ with X in Eq. (14), all terms in the kinetic term are diagonal and exhibit the canonical normalization. Moreover the field a_ϕ disappears from the potential. Note that the phase redefinition implied in U_a exactly coincides with the realization of the PQ symmetry on Φ_{12} in Eq. (12) as is to be expected (this identifies uniquely the axion degree of freedom).

Finally, the non-linear parametrization of Φ_{12} reads as

$$\Phi_{12} = \mathcal{U} M_{12} U_a, \quad (32)$$

with

$$M_{12} = \sqrt{2} \begin{pmatrix} (v+H)c_\beta - (S - i \frac{v_\phi}{\sqrt{v_\phi^2 + v^2 s_{2\beta}^2}} A_0) s_\beta & \sqrt{2} H_+ c_\beta \\ \sqrt{2} H_- s_\beta & (v+H)s_\beta + (S + i \frac{v_\phi}{\sqrt{v_\phi^2 + v^2 s_{2\beta}^2}} A_0) c_\beta \end{pmatrix} \quad (33)$$

and

$$v+H = \frac{c_\beta}{\sqrt{2}} \Re[\alpha_0] + \frac{s_\beta}{\sqrt{2}} \Re[\beta_0], \quad S = -\frac{s_\beta}{\sqrt{2}} \Re[\alpha_0] + \frac{c_\beta}{\sqrt{2}} \Re[\beta_0], \quad H_\pm = \frac{c_\beta \beta_\pm - s_\beta \alpha_\pm}{2}, \quad (34)$$

in terms of the fields in Eq. (1). The singlet field ϕ is non-linearly parametrized as

$$\phi = \left(v_\phi + \rho - i \frac{v s_{2\beta}}{\sqrt{v_\phi^2 + v^2 s_{2\beta}^2}} A_0 \right) \exp \left(i \frac{a_\phi}{\sqrt{v_\phi^2 + v^2 s_{2\beta}^2}} \right). \quad (35)$$

With the parametrizations above the kinetic term is diagonal in terms of the fields of M_{12} and ρ . Moreover, the potential is independent of the axion and Goldstone bosons. All the fields appearing in Eqs. (33) and (35) have vanishing VEVs.

Let us stress that H , S and ρ are not mass eigenstates and their relations with the h_i mass eigenstates are defined through

$$H = \sum_{i=1}^3 R_{Hi} h_i, \quad S = \sum_{i=1}^3 R_{Si} h_i, \quad \rho = \sum_{i=1}^3 R_{\rho i} h_i. \quad (36)$$

The rotation matrix R as well as the corresponding mass matrix are given in Appendix B. H and S are the so called interaction eigenstates. In particular, H couples to the gauge fields in the same way that the usual MSM Higgs does.

A. Integrating out the heavy Higgs fields

In this section we want to integrate out the heavy scalars in Φ_{12} of Eq. (32) in order to build a low-energy effective theory at the TeV scale with an axion and a light Higgs.

As a first step, let us imagine that *all* the states in Φ_{12} are heavy; upon their integration we will recover the Effective Chiral Lagrangian [15]

$$\mathcal{L} = \frac{v^2}{4} \text{Tr} D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U} + \sum_{i=0}^{13} a_i \mathcal{O}_i, \quad (37)$$

where the \mathcal{O}_i is a set of local gauge invariant operators [16], and the symbol D_μ represents the covariant derivative defined in (31). The corresponding effective couplings a_i collect the low energy information (up to energies $E \simeq 4\pi v$) pertaining to the heavy states integrated out. In the unitarity gauge, the term $D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U}$ would generate the gauge boson masses.

If a light Higgs ($h = h_1$) and axion are present, they have to be included explicitly as dynamical states [17], and the corresponding effective Lagrangian will be (gauge terms are omitted in the present discussion)

$$\begin{aligned} \mathcal{L} = & \frac{v^2}{4} \left(1 + 2g_1 \frac{h}{v} + g_2 \frac{h^2}{v^2} + \dots \right) \text{Tr} \mathcal{D}_\mu \mathcal{U}^\dagger \mathcal{D}^\mu \mathcal{U} \\ & + \left(\frac{v_\phi^2}{v_\phi^2 + v^2 s_{2\beta}} \right) \partial_\mu a_\phi \partial^\mu a_\phi + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) \\ & + \sum_{i=0}^{13} a_i \left(\frac{h}{v} \right) \mathcal{O}_i + \mathcal{L}_{\text{ren}}, \end{aligned} \quad (38)$$

where ³

$$\mathcal{D}_\mu \mathcal{U} = D_\mu \mathcal{U} + \mathcal{U} (\partial_\mu U_a) U_a^\dagger \quad (39)$$

formally amounting to a redefinition of the ‘right’ gauge field and

$$V(h) = \frac{m_h^2}{2} h^2 - d_3 (\lambda v) h^3 - d_4 \frac{\lambda}{4} h^4, \quad (40)$$

$$\mathcal{L}_{\text{ren}} = \frac{c_1}{v^4} (\partial_\mu h \partial^\mu h)^2 + \frac{c_2}{v^2} (\partial_\mu h \partial^\mu h) \text{Tr} \mathcal{D}_\nu \mathcal{U}^\dagger \mathcal{D}^\nu \mathcal{U} + \frac{c_3}{v^2} (\partial_\mu h \partial^\mu h) \text{Tr} \mathcal{D}^\mu \mathcal{U}^\dagger \mathcal{D}_\nu \mathcal{U}. \quad (41)$$

³ Note that the axion kinetic term is not well normalized in this expression yet. Extra contributions to the axion kinetic term also come from the term in the first line of Eq. (38). Only once we include these extra contributions, the axion kinetic term gets well normalized. See also discussion below.

Here h is the lightest 0^+ mass eigenstate, with mass 125 GeV but couplings in principle different from the ones of a MSM Higgs. The terms in \mathcal{L}_{ren} are required for renormalizability [18] at the one-loop level and play no role in the discussion.

The couplings a_i are now functions of h/v , $a_i(h/v)$, which are assumed to have a regular expansion and contribute to different effective vertices. Their constant parts $a_i(0)$ are related to the electroweak precision parameters ('oblique corrections').

Let us see how the previous Lagrangian (38) can be derived. First, we integrate out from $\Phi_{12} = \mathcal{U}M_{12}U_a$ all heavy degrees of freedom, such as H^\pm and A_0 , whereas we retain H and S because they contain a h_1 component, namely

$$\Phi_{12} = \mathcal{U}U_a\overline{M}_{12}, \quad \overline{M}_{12} = \sqrt{2} \begin{pmatrix} (v+H)c_\beta - Ss_\beta & 0 \\ 0 & (v+H)s_\beta + Sc_\beta \end{pmatrix}, \quad (42)$$

where H and S stand respectively for $R_{H1}h_1$ and $R_{S1}h_1$.

When the derivatives of the kinetic term of Eq. (30) act on \overline{M}_{12} , we get the contribution $\partial_\mu h \partial^\mu h$ in Eq. (38). Since the unitarity matrices, \mathcal{U} and U_a drop from the potential of Eq. (10) only $V(h)$ remains.

To derive the first line of Eq. (38), we can use Eqs. (39) and (42) to work out from the kinetic term of Eq. (30) the contribution

$$\text{Tr}(\mathcal{D}_\mu \mathcal{U} \overline{M}_{12})^\dagger \mathcal{D}^\mu \mathcal{U} \overline{M}_{12} = \frac{v^2}{4} \left(1 + 2\frac{H}{v} + \dots \right) \text{Tr} \mathcal{D}_\mu \mathcal{U}^\dagger \mathcal{D}^\mu \mathcal{U} + \mathcal{L}(a_\phi, h). \quad (43)$$

Here we used that $\overline{M}_{12}\overline{M}_{12}^\dagger$ has a piece proportional to the identity matrix and another proportional to τ_3 that cannot contribute to the coupling with the gauge bosons since $\text{Tr} D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U} \tau_3$ vanishes identically. The linear contribution in S is of this type thus decoupling from the gauge sector and as a result only terms linear in H survive. Using that $[U_a, \overline{M}_{12}] = [U_a, \tau_3] = 0$, the matrix U_a cancels out in all traces and the only remains of the axion in the low energy action is the modification $D_\mu \rightarrow \mathcal{D}_\mu$. The resulting effective action is invariant under global transformations $\mathcal{U} \rightarrow L\mathcal{U}R^\dagger$ but now R is an $SU(2)$ matrix only if custodial symmetry is preserved (i.e. $\tan\beta = 1$). Otherwise the right global symmetry group is reduced to the $U(1)$ subgroup. It commutes with $U(1)_{PQ}$.

We then reproduce (38) with $g_1 = 1$. However, this is true for the field H on the l.h.s. of Eq. (43), not $h = h_1$ and this will reflect in a reduction in the value of the g_i when one considers the coupling to the lightest Higgs only.

A coupling among the S field, the axion and the neutral Goldstone or the neutral gauge boson survives in Eq. (43). This will be discussed in Section V. As for the axion kinetic term, it is reconstructed with the proper normalization from the first term in (30) together with a contribution from the ‘connection’ $(\partial_\mu U_a)U_a^\dagger$ in $\text{Tr } \mathcal{D}_\mu \mathcal{U} \mathcal{D}^\mu \mathcal{U}$ (see Eq. (52) in next section). There are terms involving two axions and the Higgs that are not very relevant phenomenologically at this point. This completes the derivation of the $\mathcal{O}(p^2)$ terms in the effective Lagrangian.

To go beyond this tree level and to determine the low energy constants $a_i(0)$ in particular requires a one-loop integration of the heavy degrees of freedom and matching the Green’s functions of the fundamental and the effective theories.

See e.g. [15, 16] for a classification of all possible operators appearing up to $\mathcal{O}(p^6)$ that are generated in this process. The information on physics beyond the MSM is encoded in the coefficients of the effective chiral Lagrangian operators. Without including the (lightest) Higgs field h (i.e. retaining only the constant term in the functions $a_i(h/v)$) and ignoring the axion, there are only two independent $\mathcal{O}(p^2)$ operators

$$\mathcal{L}^2 = \frac{v^2}{4} \text{Tr}(D_\mu \mathcal{U} D^\mu \mathcal{U}^\dagger) + a_0(0) \frac{v^2}{4} (\text{Tr}(\tau^3 \mathcal{U}^\dagger D_\mu \mathcal{U}))^2 \quad (44)$$

The first one is universal, its coefficient being fixed by the W mass. As we just saw it is flawlessly reproduced in the 2HDM at tree level after assuming that the additional degrees of freedom are heavy. Loop corrections do not modify it if v is the physical Fermi scale. The other one is related to the ρ parameter. In addition there are a few $\mathcal{O}(p^4)$ operators with their corresponding coefficients

$$\mathcal{L}^4 = \frac{1}{2} a_1(0) g g' \text{Tr}(\mathcal{U} B_{\mu\nu} \mathcal{U}^\dagger W^{\mu\nu}) - \frac{1}{4} a_8(0) g^2 \text{Tr}(\mathcal{U} \tau^3 \mathcal{U}^\dagger W_{\mu\nu}) \text{Tr}(\mathcal{U} \tau^3 \mathcal{U}^\dagger W^{\mu\nu}) + \dots \quad (45)$$

In the above expression $W_{\mu\nu}$ and $B_{\mu\nu}$ are the field strength tensors associated to the $SU(2)_L$ and $U(1)_Y$ gauge fields, respectively. In this paper we shall only consider the self-energy, or oblique, corrections, which are dominant in the 2HDM model just as they are in the MSM.

The oblique corrections are often parametrized in terms of the parameters ε_1 , ε_2 and ε_3 introduced in [19]. In an effective theory such as the one described by the Lagrangian (44) and (45) $\varepsilon_1, \varepsilon_2$ and ε_3 receive one loop (universal) contributions from the leading $\mathcal{O}(p^2)$ term $v^2 \text{Tr}(D_\mu \mathcal{U} D^\mu \mathcal{U}^\dagger)$ and tree level contributions from the $a_i(0)$. Thus

$$\varepsilon_1 = 2a_0(0) + \dots \quad \varepsilon_2 = -g^2 a_8(0) + \dots \quad \varepsilon_3 = -g^2 a_1(0) + \dots \quad (46)$$

where the dots symbolize the one-loop $\mathcal{O}(p^2)$ contributions. The latter are totally independent of the specific symmetry breaking sector. See e.g. [12] for more details.

A systematic integration of the heavy degrees of freedom, including the lightest Higgs as external legs, would provide the dependence of the low-energy coefficient functions on h/v , i.e. the form of the functions $a_i(h/v)$. However this is of no interest to us here.

V. HIGGS AND AXION EFFECTIVE COUPLINGS

The coupling of h_1 can be worked out from the one of H , which is exactly as in the MSM, namely

$$g_1^{SM} H W_\mu W^\mu = g_1^{SM} (R_{H1} h_1 + R_{H2} h_2 + R_{H3} h_3) W_\mu W^\mu \quad (47)$$

where $R_{H1} = 1 - (v/v_\phi)^2 A_{13}^2/2$ and $g_1^{SM} \equiv 1$. With the expression of A_{13} given in Appendix B,

$$g_1 = g_1^{SM} \times \left(1 - \frac{2v^2}{v_\phi^2 \lambda_\phi^2} (ac_\beta^2 + bs_\beta^2 c_{2\beta} - cs_{2\beta})^2 \right). \quad (48)$$

It is clear that in cases 1 to 3 the correction to the lightest Higgs couplings to the gauge bosons are very small, experimentally indistinguishable from the MSM case. In any case the correction is negative and $g_1 < g_1^{SM}$.

Case 4 falls in a different category. Let us remember that this case corresponds to the situation where $c \sim \lambda_i v^2/v_\phi^2$. Then the corresponding rotation matrix is effectively 2×2 , with an angle θ that is given in Appendix B. Then

$$g_1 = g_1^{SM} \cos \theta. \quad (49)$$

In the custodial limit, $\lambda_1 = \lambda_2$ and $\tan \beta = 1$, this angle vanishes exactly and $g_1 = g_1^{SM}$. Otherwise this angle could have any value. Note however that when $c \gg \lambda_i v^2/v_\phi^2$ then $\theta \rightarrow 0$ and the value $g_1 \simeq g_1^{SM}$ is recovered. This is expected as when c grows case 4 moves into case 3. Experimentally, from the LHC results we know [20] that $g_1 = [0.67, 1.25]$ at 95% CL.

Let us now discuss the Higgs-photon-photon coupling in this type of models. Let us first consider the contribution from gauge and scalar fields in the loop. The diagrams contributing to the coupling between the lightest scalar state h_1 and photons are exactly the same ones as in a generic 2HDM, via a loop of gauge bosons and one of charged Higgses. In the DFSZ

case the only change with respect to a generic 2HDM could be a modification in the $h_1 WW$ (or Higgs-Goldstone bosons coupling) or in the $h_1 H^+ H^-$ tree-level couplings. The former has already been discussed while the triple coupling of the lightest Higgs to two charged Higgses gets modified in the DFSZ model to

$$\begin{aligned} \lambda_{h_1 H^+ H^-} = & 8vR_{H1} [(\lambda_1 + \lambda_2)s_{2\beta}^2 + 4\lambda_3 + 2\lambda_4] + 16vs_{2\beta}R_{S1} (\lambda_2 c_\beta^2 - \lambda_1 s_\beta^2) \\ & + 8v_\phi R_{\rho 1} (as_\beta^2 + bc_\beta^2 - cs_{2\beta}). \end{aligned} \quad (50)$$

Note that the first line involves only constants that are already present in a generic 2HDM, while the second one does involve the couplings a, b and c characteristic of the DFSZ model.

The coupling of the lightest Higgs to the up and down quarks is obtained from the Yukawa terms in Eq. (2)

$$\mathcal{L}(u, d, h_1) = \sqrt{2}h_1 [G_1 (R_{H1}c_\beta - R_{S1}s_\beta) \bar{u}_L u_R + G_2 (R_{H1}s_\beta + R_{S1}c_\beta) \bar{d}_L d_R + \text{h.c.}] \quad (51)$$

The corresponding entries of the rotation matrix in the 0^+ sector can be found in Appendix B. In cases 1, 2 and 3 the relevant entries are $R_{H1} \sim 1$, $R_{S1} \sim v^2/v_\phi^2$ and $R_{\rho 1} \sim v/v_\phi$, respectively. Therefore the second term in the first line is always negligible but the piece in the second one can give a sizeable contribution if c is of $\mathcal{O}(1)$ (case 1). This case could therefore be excluded or confirmed from a precise determination of this coupling. In cases 2 and 3 this effective coupling aligns itself with a generic 2HDM but with large (typically ~ 100 TeV) or moderately large (few TeV) charged Higgs masses.

Case 4 is slightly different again. In this case $R_{H1} = \cos \theta$ and $R_{S1} = \sin \theta$ but $R_{\rho 1} = 0$. The situation is again similar to a generic 2HDM, now with masses that can be made relatively light, but with a mixing angle that because of the presence of the c in (B9) terms may differ slightly from the 2HDM. For a review of current experimental fits in 2HDM the interested reader can see [11].

In this section we will also list the tree-level couplings of the axion to the light fields, thus completing the derivation of the effective low energy theory. The tree-level couplings are very few actually as the axion does not appear in the potential, and they are necessarily derivative in the bosonic part. From the kinetic term we get

$$\mathcal{L}(a_\phi, h) = \frac{2R_{S1}}{\sqrt{v_\phi^2 + v^2 s_{2\beta}^2}} h_1 \partial^\mu a_\phi (\partial_\mu G_0 + m_Z Z_\mu) + \text{terms with 2 axions}, \quad (52)$$

From the Yukawa terms (2) we also get

$$\mathcal{L}(a_\phi, q, \bar{q}) = \frac{2i}{\sqrt{v_\phi^2 + v^2 s_{2\beta}^2}} a_\phi (m_u s_\beta^2 \bar{u} \gamma_5 u + m_d c_\beta^2 \bar{d} \gamma_5 d). \quad (53)$$

The loop-induced couplings between the axion and gauge bosons (such as the anomaly-induced coupling $a_\phi \tilde{F}F$, of extreme importance for direct axion detection [14]) will not be discussed here as they are amply reported in the literature.

VI. MATCHING THE DFSZ MODEL TO 2HDM

The most general 2HDM potential can be read ⁴ e.g. from from [10, 11]

$$\begin{aligned} V(\phi_1, \phi_2) = & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - \left[m_{12}^2 \phi_1^\dagger \phi_2 + \text{h.c.} \right] \\ & + \frac{\Lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\Lambda_2}{2} (\phi_2^\dagger \phi_2)^2 + \Lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \Lambda_4 |\phi_1^\dagger \phi_2|^2 \\ & + \left[\frac{\Lambda_5}{2} (\phi_1^\dagger \phi_2)^2 + \Lambda_6 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_2) + \Lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + \text{h.c.} \right]. \end{aligned} \quad (54)$$

This potential contains 4 complex and 6 real parameters (i.e. 14 real numbers). The most popular 2HDM is obtained by imposing a Z_2 symmetry that is softly broken; namely $\Lambda_6 = \Lambda_7 = 0$ and $m_{12} \neq 0$. The Z_2 approximate invariance helps remove flavour changing neutral current at tree-level. A special role is played by the term proportional to m_{12} . This term softly breaks Z_2 but is necessary to control the decoupling limit of the additional scalars in a 2HDM and to eventually reproduce the MSM with a light Higgs.

In the DFSZ model discussed here v_ϕ is very large and at low energies the additional singlet field ϕ reduces approximately to $\phi \simeq v_\phi \exp(a_\phi/v_\phi)$. Indeed, from (35) we see that ϕ has a A_0 component but it can be in practice neglected for an invisible axion since this component is $\propto v/v_\phi$. In addition the radial variable ρ can be safely integrated out.

Thus, the low-energy effective theory defined by the DFSZ model is a particular type of 2HDM model with the non-trivial benefit of solving the strong CP problem thanks to the appearance of an invisible axion⁵. Indeed DFSZ reduces at low energy to a 2HDM containing

⁴ We have relabelled $\lambda \rightarrow \Lambda$ to avoid confusion with the potential of the DFSZ model.

⁵ Recall that mass generation due to the anomalous coupling with gluons has not been considered in this work

9 parameters in practice (see below, note that v_ϕ is used as input) instead of the 14 of the general 2HDM case.

The constants $\Lambda_{6,7}$ are absent as in many Z_2 invariant 2HDM but also $\Lambda_5 = 0$. All these terms are not invariant under the Peccei-Quinn symmetry. In addition the m_{12} that softly breaks Z_2 and is necessary to control the decoupling to the MSM is dynamically generated by the PQ spontaneously symmetry breaking. There is no $\mu = m_{12}$ problem here concerning the naturalness of having non-vanishing μ .

At the electroweak scale the DFSZ potential of eq. (6) can be matched to the 2HDM terms of (54) by the substitutions

$$m_{11}^2 = [-2\lambda_1 V_1^2 + 2\lambda_3(V_1^2 + V_2^2) + av_\phi^2] / 4 \quad (55)$$

$$m_{22}^2 = [-2\lambda_1 V_2^2 + 2\lambda_3(V_1^2 + V_2^2) + bv_\phi^2] / 4 \quad (56)$$

$$m_{12}^2 = cv_\phi^2 / 4 \quad (57)$$

$$\Lambda_1 = (\lambda_1 + \lambda_3)/8, \quad \Lambda_2 = (\lambda_2 + \lambda_3)/8 \quad (58)$$

$$\Lambda_3 = (2\lambda_3 + \lambda_4)/16, \quad \Lambda_4 = -\lambda_4/16, \quad (59)$$

$$\Lambda_5 = 0, \quad \Lambda_6 = 0, \quad \Lambda_7 = 0 \quad (60)$$

Combinations of parameters of the DFSZ potential can be determined from the four masses m_{h_1} , m_{h_2} , m_{A_0} and m_{H^\pm} and the two parameters g_1 (or θ) and $\lambda_{h_1 H^+ H^-}$ that controls the Higgs- WW and (indirectly) the Higgs- $\gamma\gamma$ couplings, whose expression in terms of the parameters of the potential has been given. As we have seen for generic couplings, all masses but the lightest Higgs decouple and the effective couplings take their MSM values. In the phenomenologically more interesting cases (cases 3 and 4) two of the remaining constants (a , b) drop in practice from the low-energy predictions and the effective 2HDM corresponding to DFSZ depends only on 7 parameters. If in addition custodial symmetry is assumed to be exact or nearly exact, the relevant parameters are actually totally determined by measuring three masses and the two couplings (m_{h_2} turns out to be equal to m_{H^\pm} if custodial invariance holds). Therefore LHC has the potential of fully determining all the relevant parameters of the DFSZ model.

Eventually the LHC and perhaps a LC will be hopefully able to assess the parameters of the 2HDM potential and their symmetries to check the DFSZ relations. Of course finding a pattern of couplings in concordance with the pattern predicted by the low energy limit

of DFSZ model would not yet prove the latter to be the correct microscopic theory as this would require measuring the axion couplings, which are not present in a 2HDM. In any case, it should be obvious that the effective theory of the DFSZ is significantly more restrictive than a general 2HDM.

We emphasize that the above discussion refers mostly to case 4 as discussed in this work and it partly applies to case 3 too. Cases 1 and 2 are in practice indistinguishable from the MSM up to energies that are substantially larger from the ones currently accessible, apart from the presence of the axion itself. As we have seen, the DFSZ in this case is quite predictive and it does not correspond to a generic 2HDM but to one where massive scalars are all decoupled with the exception of the 125 GeV Higgs.

VII. CONSTRAINTS FROM ELECTROWEAK PARAMETERS

For the purposes of setting bounds on the masses of the new scalars in the 2HDM, $\varepsilon_1 = \Delta\rho$ is the most effective one. For this reason we will postpone the analysis of ε_2 and ε_3 to a future publication.

ε_1 can be computed by [19]

$$\varepsilon_1 \equiv \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}, \quad (61)$$

with the gauge boson vacuum polarization functions defined as

$$\Pi_{VV}^{\mu\nu}(q) = g^{\mu\nu}\Pi_{VV}(q^2) + q^\mu q^\nu \text{ terms} \quad (V = W, Z). \quad (62)$$

We need to compute loops of the type of Fig. 2. These diagrams produce three kinds of

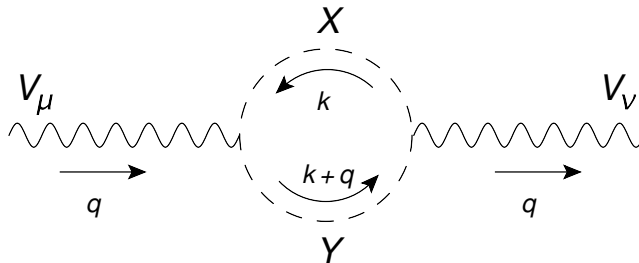


FIG. 2: Feynman diagram relevant for $\Pi_{VV}^{\mu\nu}(q)$.

terms. The terms proportional to two powers of the external momentum, $q_\mu q_\nu$, do not enter

in $\Pi_{VV}(q^2)$. The terms proportional to just one power vanish upon integration. Only the terms proportional to $k_\mu k_\nu$ survive and contribute.

Although it is an unessential approximation, to keep formulae relatively simple we will compute ε_1 in the approximation $g' = 0$. The term proportional to $(g')^2$ is actually the largest contribution in the MSM (leaving aside the breaking due to the Yukawa couplings) but it is only logarithmically dependent on the masses of any putative scalar state and it can be safely omitted for our purposes [12]. The underlying reason is that in the 2HDM custodial symmetry is ‘optional’ in the scalar sector and it is natural to investigate power-like contributions that would provide the strongest constraints. We obtain, in terms of the mass eigenstates and the rotation matrix of Eq. (36),

$$\varepsilon_1 = \frac{1}{16\pi^2 v^2} \left[m_{H^\pm}^2 - \frac{v_\phi^2}{v_\phi^2 + v^2 s_{2\beta}^2} f(m_{H^\pm}^2, m_{A_0}^2) \right. \quad (63)$$

$$\left. + \sum_{i=1}^3 R_{Si}^2 \left(\frac{v_\phi^2}{v_\phi^2 + v^2 s_{2\beta}^2} f(m_{A_0}^2, m_{h_i}^2) - f(m_{H^\pm}^2, m_{h_i}^2) \right) \right], \quad (64)$$

where $f(a, b) = ab/(b-a) \log b/a$ and $f(a, a) = a$. Setting $v_\phi \rightarrow \infty$ and keeping Higgs masses fixed, we formally recover the $\Delta\rho$ expression in the 2HDM (see the Appendix in [12]), namely

$$\varepsilon_1 = \frac{1}{16\pi^2 v^2} \left[m_{H^\pm}^2 - f(m_{H^\pm}^2, m_{A_0}^2) + \sum_{i=1}^3 R_{Si}^2 (f(m_{A_0}^2, m_{h_i}^2) - f(m_{H^\pm}^2, m_{h_i}^2)) \right]. \quad (65)$$

Now, in the limit $v_\phi \rightarrow \infty$ and $m_{H^\pm} \rightarrow m_{A_0}$ (cases 1, 2 or 3 previously discussed) the $\Delta\rho$ above will go to zero as v/v_ϕ at least and the experimental bound is fulfilled automatically.

However, we are particularly interested in case 4 that allows for a light spectrum of new scalar states. We will study this in two steps. First we assume a ‘quasi-custodial’ setting whereby we assume that custodial symmetry is broken *only* via the coupling $\lambda_{4B} = \lambda_4 - 2\lambda$ being non-zero. Imposing vacuum stability (see e.g. Appendix D) and the experimental bound of $(\varepsilon_1 - \varepsilon_1^{SM})/\alpha = \Delta T = 0.08(7)$ from the electroweak fits in [21] one gets the exclusion plots shown in Fig. 3.

It is also interesting to show (in this same ‘quasi-custodial’ limit) the range of masses allowed by the present constraints on ΔT , without any reference to the parameters in the potential. This is shown for two reference values of m_{A_0} in Fig. 4. Note the severe constraints due to the requirement of vacuum stability.

Finally let us turn to the consideration of the general case 4. We now completely give up custodial symmetry and hence the three masses m_{A_0} , m_{H^\pm} and m_{h_2} are unrelated, except

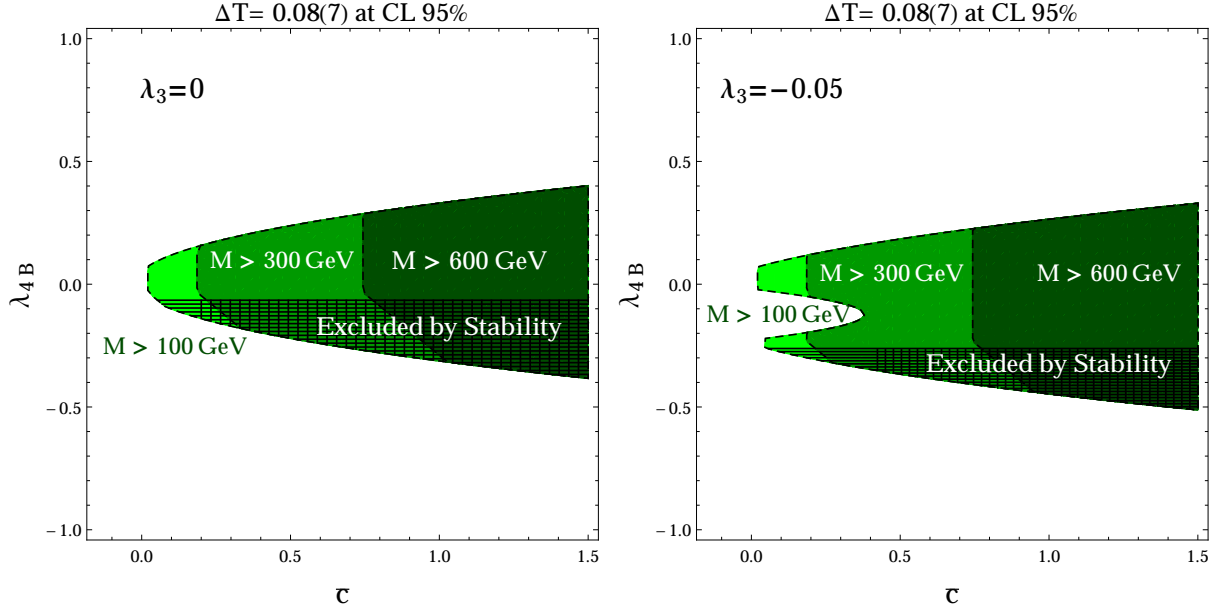


FIG. 3: Exclusion region for a ‘quasi-custodial’ 2HDM potential with a $\lambda_{4B} = \lambda_4 - 2\lambda$ custodial breaking term. In this limit, the masses depend on the free parameters $\lambda_{4B} = \lambda_4 - 2\lambda$, λ_3 and $\bar{c} = cv_\phi^2/v^2$, and then the vacuum stability conditions of Appendix D and ΔT can be used to exclude regions of the free parameters above. The left, right plots are for $\lambda_3 = -0.05$, $\lambda_3 = 0$, respectively. Different color regions imply different cuts assuming that all masses (m_{A_0} , m_{H_\pm} and m_{h_2}) are greater than 100, 300 or 600 GeV (light to dark). The potential becomes unstable for $\lambda_3 > 0.03$.

for the eventual lack of stability of the potential. In this case, the rotation R can be different from the identity which was the case in the ‘quasi-custodial’ scenario above. In particular, $R_{S2} = \cos \theta$ from Appendix B and the angle θ is not vanishing. However, experimentally $\cos \theta$ is known to be very close to one (see section V). If we assume that $\cos \theta$ is exactly equal to one, we get the exclusion/acceptance regions shown in Fig. 5. Finally, Fig. 6 depicts the analogous plot for $\cos \theta = 0.95$ that is still allowed by existing constraints. We see that the allowed range of masses are much more severely restricted in this case.

VIII. CONCLUSIONS

With the LHC experiments gathering more data, the exploration of the symmetry breaking sector of the Standard Model will gain renewed impetus. Likewise, it is important to

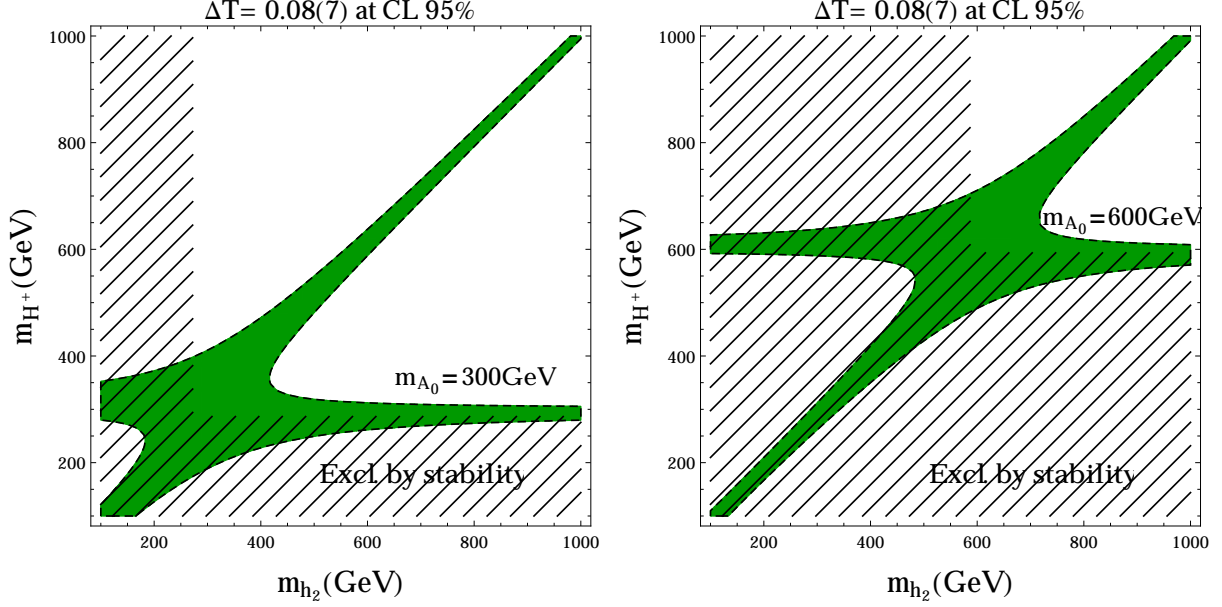


FIG. 4: Exclusion plot imposed by the constraint from ΔT on the second 0^+ state (i.e. ‘second Higgs’) and the charged Higgs masses for two reference values of M_{A_0} (left: 300 GeV, right: 600 GeV) in the ‘quasi-custodial’ case discussed in Fig. 3. The concentration of points along approximately two axis is easy to understand after inspection of the relevant formula for ΔT . ΔT is vanishing at the custodial limit $M_{h_2} = M_{H^\pm}$, and also for $M_{H^\pm} = M_{A_0}$. The regions excluded by considerations of stability of the potential are shown.

search for dark matter candidates as this is a degree of freedom certainly missing in the minimal Standard Model. An invisible axion is an interesting candidate for dark matter; however trying to look for direct evidence of its existence at the LHC is hopeless as it is extremely weakly coupled. Therefore we have to resort to less direct ways to explore this sector by formulating consistent models that include the axion and deriving consequences that could be experimentally tested.

In this work we have explored such consequences in the DFSZ model, an extension of the popular 2HDM. A necessary characteristic of models with an invisible axion is the presence of the Peccei-Quinn symmetry. This restricts the form of the effective potential. We have taken into account the recent data on the Higgs mass and several effective couplings, and included the constraints from electroweak precision parameters.

Four possible scenarios have been considered. In virtually all points of parameter space of the DFSZ model we do not really expect to see any relevant modifications with respect

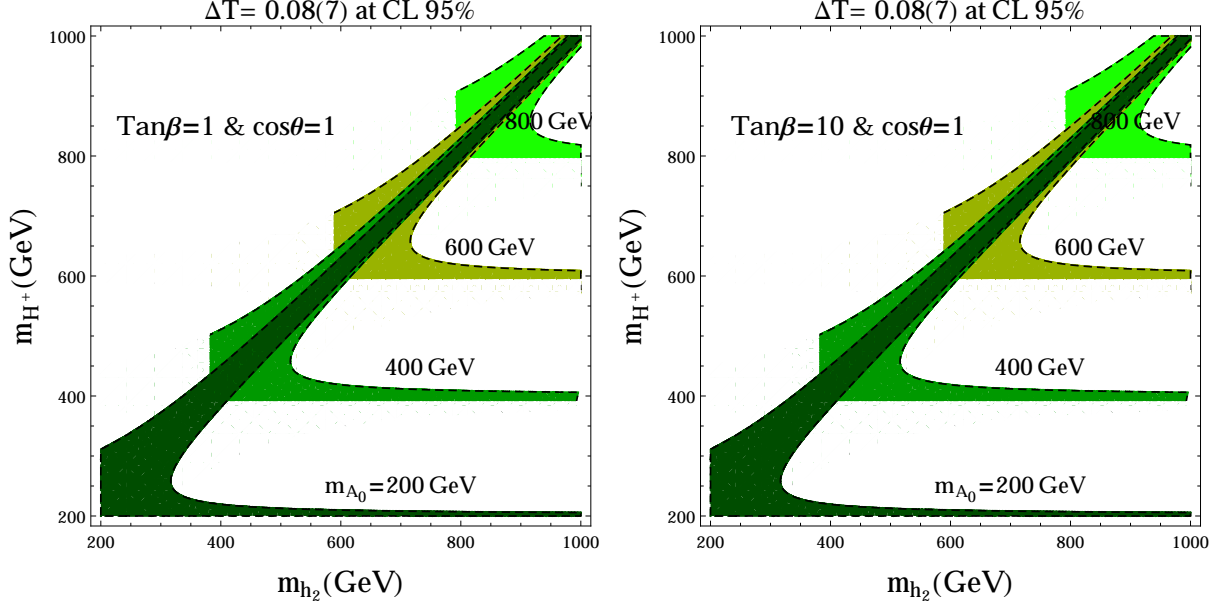


FIG. 5: Exclusion plot imposed by the constraint from ΔT on the second 0^+ state (i.e. ‘second Higgs’) and the charged Higgs masses for several reference values of m_{A_0} and $\tan\beta$ in the general case. The value $\cos\theta = 1$ is assumed here. The successive horizontal bands correspond to different values of m_{A_0} . The stability bounds have already been implemented, effectively cutting off the left and lower arms of the regions otherwise acceptable.

to the minimal Standard Model predictions. The new scalars have masses of order v_ϕ or $\sqrt{vv_\phi}$ in two of the cases discussed. The latter could perhaps be reachable with a 100 TeV circular collider although this is not totally guaranteed. In a third case, it would be possible to get scalars in the multi-TeV region, making this case testable in the future at the LHC. Finally, we have identified a fourth situation where a relatively light spectrum emerges. The last two cases correspond to a situation where the coupling between the singlet and the two doublets is of order v^2/v_ϕ^2 ; i.e. very small (10^{-10} or less) and in order to get a relatively light spectrum in addition one has to require some couplings to be commensurate (but not necessarily fine-tuned).

The fact that some specific couplings are required to be very small may seem odd, but as it has been argued elsewhere it is technically natural, as the couplings in question do break some extended symmetry and are therefore protected. From this point of view these values are perfectly acceptable.

The results on the scalar spectrum are derived here at tree level only and are of course

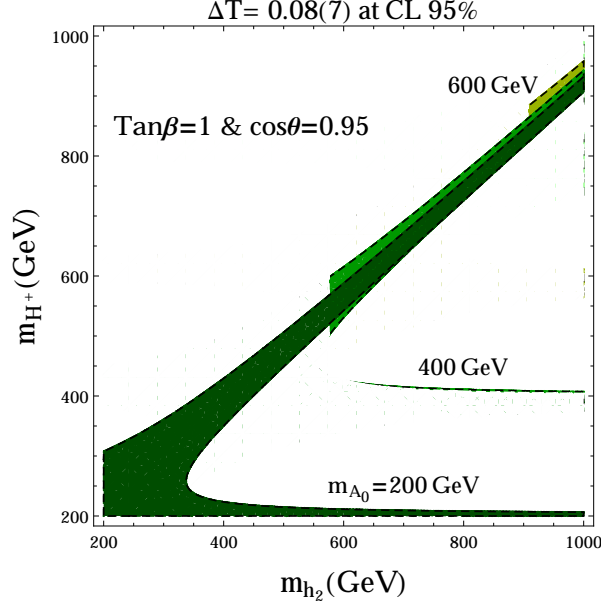


FIG. 6: Exclusion plot imposed by the constraint from ΔT on the second 0^+ state (i.e. ‘second Higgs’) and the charged Higgs masses for several reference values of m_{A_0} . Here we take $\tan \beta = 1$ and allow $\cos \theta = 0.95$, which is consistent with present constraints.

subject to large radiative corrections in principle. However one should note two ingredients that should ameliorate the hierarchy problem. The first observation is that the mass of the 0^- scalar is directly proportional to c ; it is exactly zero if the additional symmetries discussed in [7] hold. It is therefore somehow protected. On the other hand custodial symmetry relates different masses, helping to maintain other relations. Some hierarchy problem should still remain but of a magnitude very similar to the one already present in the minimal Standard Model.

We have imposed on the model known constraints such as the fulfilment of the bounds on the ρ -parameter. These bounds turn out to be automatically fulfilled in most of parameter space and become only relevant when the spectrum is light (case 4). This is particularly relevant as custodial symmetry is by no means automatic in the 2HDM. Somehow the introduction of the axion and the related Peccei-Quinn symmetry makes possible custodially violating consequences naturally small. We have also considered the experimental bounds on the Higgs-gauge bosons and Higgs-two photons couplings. Together with four scalar masses, these parameters determine in an almost unique way the DFSZ potential, thus showing that it has substantial less room to maneuver than a generic 2HDM.

In conclusion, DFSZ models containing an invisible axion are natural and, in spite of the large scale that appears in the model to make the axion nearly invisible, there is the possibility that they lead to a spectrum that can be tested at the LHC. This spectrum is severely constrained, making it easier to prove or disprove such possibility in the near future. On the other hand it is perhaps more likely that the new states predicted by the model lie beyond the LHC range. In this situation the model hides itself by making indirect contributions to most observables quite small.

Acknowledgements

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Appendix A: Minimization conditions of the potential

The minimization conditions for the potential (6) are

$$\lambda_1 (2v^2 c_\beta^2 - V_1^2) + \lambda_3 (2v^2 - V_1^2 - V_2^2) + \frac{v_\phi^2}{2} (a + c \tan \beta) = 0, \quad (\text{A1})$$

$$\lambda_2 (2v^2 s_\beta^2 - V_2^2) + \lambda_3 (2v^2 - V_1^2 - V_2^2) + \frac{v_\phi^2}{2} \left(b + \frac{c}{\tan \beta} \right) = 0, \quad (\text{A2})$$

$$\lambda_\phi (v_\phi^2 - V_\phi^2) + 2v^2 (ac_\beta^2 + bs_\beta^2 - cs_{2\beta}) = 0. \quad (\text{A3})$$

These allow us to eliminate the dimensionful parameters V_ϕ , V_1 and V_2 in favor of the different couplings, v and v_ϕ . In the case where $\lambda_\phi = 0$ it is also possible to eliminate c .

Appendix B: 0^+ neutral sector mass matrix

The 3×3 mass matrix is

$$M_{HS\rho} = 4 \begin{pmatrix} 8v^2 (\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_3) & 4v^2 (-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2) s_{2\beta} & 2vv_\phi (ac_\beta^2 + bs_\beta^2 - cs_{2\beta}) \\ 4v^2 (-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2) s_{2\beta} & \frac{2cv_\phi^2}{s_{2\beta}} + 2v^2 (\lambda_1 + \lambda_2) s_{2\beta}^2 & -vv_\phi [(a-b) s_{2\beta} + 2cc_{2\beta}] \\ 2vv_\phi (ac_\beta^2 + bs_\beta^2 - cs_{2\beta}) & -vv_\phi [(a-b) s_{2\beta} + 2cc_{2\beta}] & \lambda_\phi v_\phi^2 \end{pmatrix} \quad (\text{B1})$$

This is diagonalized with a rotation

$$\begin{pmatrix} H \\ S \\ \rho \end{pmatrix} = R \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}. \quad (\text{B2})$$

We write the rotation matrix as

$$R = \exp \left(\frac{v}{v_\phi} A + \frac{v^2}{v_\phi^2} B \right), \quad A^T = -A, \quad B^T = -B \quad (\text{B3})$$

and work up to second order in v/v_ϕ . We find

$$A_{12} = B_{13} = B_{23} = 0, \quad (\text{B4})$$

so the matrix is

$$R = \begin{pmatrix} 1 - \frac{v^2}{v_\phi^2} \frac{A_{13}^2}{2} & -\frac{v^2}{v_\phi^2} \frac{A_{13}A_{23} - 2B_{12}}{2} & \frac{v}{v_\phi} A_{13} \\ -\frac{v^2}{v_\phi^2} \frac{A_{13}A_{23} + 2B_{12}}{2} & 1 - \frac{v^2}{v_\phi^2} \frac{A_{23}^2}{2} & \frac{v}{v_\phi} A_{23} \\ -\frac{v}{v_\phi} A_{13} & -\frac{v}{v_\phi} A_{23} & 1 - \frac{v^2}{v_\phi^2} \frac{A_{13}^2 + A_{23}^2}{2} \end{pmatrix}, \quad (\text{B5})$$

with

$$A_{13} = \frac{2}{\lambda_\phi} (ac_\beta^2 + bs_\beta^2 - cs_{2\beta}), \quad A_{23} = \frac{(a-b)s_{2\beta} + 2cc_{2\beta}}{\frac{2c}{s_{2\beta}} - \lambda_\phi}, \quad (\text{B6})$$

$$B_{12} = -\frac{2}{c} s_{2\beta}^2 (\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \frac{s_{2\beta}}{\lambda_\phi c} \frac{c - \lambda_\phi s_{2\beta}}{2c - \lambda_\phi s_{2\beta}} (ac_\beta^2 + bs_\beta^2 - cs_{2\beta}) [(a-b)s_{2\beta} + 2cc_{2\beta}] \quad (\text{B7})$$

In the case of section III when the breaking of custodial symmetry is $SU(2) \times SU(2) \rightarrow U(1)$ the mass matrix is

$$M_{HS\rho} = 4 \begin{pmatrix} 8v^2 [\lambda_3 + \lambda(s_\beta^4 + c_\beta^4)] & -2\lambda v^2 s_{4\beta} & 2vv_\phi \left(a + 2\lambda \frac{v^2}{v_\phi^2} s_{2\beta}^2 \right) \\ -2\lambda v^2 s_{4\beta} & -4\lambda v^2 c_{2\beta}^2 & 2\lambda \frac{v^3}{v_\phi} s_{4\beta} \\ 2vv_\phi \left(a + 2\lambda \frac{v^2}{v_\phi^2} s_{2\beta}^2 \right) & 2\lambda \frac{v^3}{v_\phi} s_{4\beta} & \lambda v_\phi^2 \end{pmatrix}. \quad (\text{B8})$$

For case 4 of section III the rotation matrix is

$$R = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \tan 2\theta = -\frac{(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) s_{2\beta}}{(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) c_{2\beta} + \lambda_3 - c v_\phi^2 / (4 v^2 s_{2\beta})}. \quad (\text{B9})$$

Appendix C: The limit $\lambda_\phi = 0$

The eigenvalues of the 3×3 mass matrix in the 0^+ sector are

$$\begin{aligned} m_{h_1}^2 &= 32v^2 (\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_3) \\ m_{h_2}^2 &= \frac{v_\phi^2}{2} s_{2\beta}^2 (a c_\beta^2 + b s_\beta^2) + \mathcal{O}(v^2) \\ m_{h_3}^2 &= -8v^2 \frac{(a c_\beta - b s_\beta)^2}{a c_\beta^2 + b s_\beta^2} \end{aligned} \quad (\text{C1})$$

Either m_2^2 or m_3^2 is negative. Note that the limit of a, b small can not be taken directly in this case.

Appendix D: Vacuum stability conditions and mass relations

Vacuum stability implies the following conditions on the parameters of the potential [10]:

$$\begin{aligned} \lambda_1 + \lambda_3 &> 0, \quad \lambda_2 + \lambda_3 > 0, \quad 2\lambda_3 + \lambda_4 + 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} > 0, \\ \lambda_3 + \sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} &> 0. \end{aligned} \quad (\text{D1})$$

In the case of custodial symmetry except for $\lambda_{4B} \neq 0$, these conditions reduce to

$$\lambda + \lambda_3 > 0, \quad \lambda + 2\lambda_3 > 0, \quad 4\lambda + 4\lambda_3 + \lambda_{4B} > 0 \quad (\text{D2})$$

and assuming a, b, c very small (e.g. case 4) they impose two conditions on the masses for :

$$m_{A_0}^2 + m_{h_1}^2 - m_{h_2}^2 > 0, \quad m_{H_\pm}^2 + m_{h_1}^2 - m_{A_0}^2 > 0. \quad (\text{D3})$$

Appendix E: Vertices and Feynman Rules in the DFSZ model

In the limit of $g' = 0$, all the diagrams involved in the calculation of ε_1 are of the type of Fig. 2. All the relevant vertices are of the type seen in Fig. 7, with all momenta assumed to be incoming. The relevant Feynman rules are as follows:

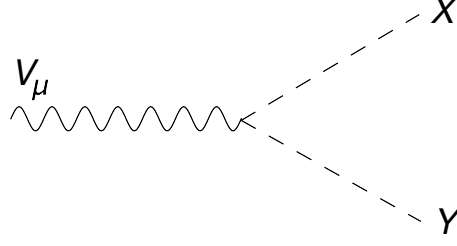


FIG. 7: Two scalars and a gauge boson.

Interaction term	Feynman Rule for the vertex
$\lambda V^\alpha X \partial_\alpha Y$	λp_Y^μ
$\lambda V^\alpha X \overset{\leftrightarrow}{\partial}_\alpha Y$	$\lambda(p_Y - p_X)^\mu$

To compute Π_{ZZ} entering Eq. (61), we need diagrams like Fig. 2 with $V = Z$. The X, Y pairs are

X	Y	Interaction term	Feynman Rule for the vertices
H_+	H_-	$-\frac{i}{2}gW_3^\alpha H_+ \overset{\leftrightarrow}{\partial}_\alpha H_-$	$\frac{g^2}{4}(2p+q)_\mu(2p+q)_\nu$
S	A_0	$\frac{g}{2}\frac{v_\phi}{\sqrt{v_\phi^2+v^2s_{2\beta}^2}}W_3^\alpha S \overset{\leftrightarrow}{\partial}_\alpha A_0$	$-\frac{g^2}{4}\frac{v_\phi^2}{v_\phi^2+v^2s_{2\beta}^2}(2p+q)_\mu(2p+q)_\nu$
S	a_ϕ	$g\frac{v\sin 2\beta}{\sqrt{v_\phi^2+v^2s_{2\beta}^2}}W_3^\alpha S \partial_\alpha a_\phi$	$-g^2\frac{v^2\sin^2 2\beta}{v_\phi^2+v^2s_{2\beta}^2}(p+q)_\mu(p+q)_\nu$
H	G_0	$-gW_3^\alpha H \partial_\alpha G_0$	$-g^2(p+q)_\mu(p+q)_\nu$
G_+	G_-	$\frac{i}{2}gW_3^\alpha G_+ \overset{\leftrightarrow}{\partial}_\alpha G_-$	$\frac{g^2}{4}(2p+q)_\mu(2p+q)_\nu$

To compute Π_{WW} entering Eq. (61), we need diagrams like Fig. 2 with $V = W_+$. The X, Y pairs are

X	Y	Interaction term	Feynman Rule for the vertices
H_+	S	$\frac{i}{2}gW_+^\alpha H_- \overset{\leftrightarrow}{\partial}_\alpha S$	$\frac{g^2}{4}(2p+q)_\mu(2p+q)_\nu$
H_+	A_0	$\frac{g}{2}\frac{v_\phi}{\sqrt{v_\phi^2+v^2s_{2\beta}^2}}W_+^\alpha A_0 \overset{\leftrightarrow}{\partial}_\alpha H_-$	$-\frac{g^2}{4}\frac{v_\phi^2}{v_\phi^2+v^2s_{2\beta}^2}(2p+q)_\mu(2p+q)_\nu$
H_+	a_ϕ	$-g\frac{v\sin 2\beta}{\sqrt{v_\phi^2+v^2s_{2\beta}^2}}W_+^\alpha H_- \partial_\alpha a_\phi$	$-g^2\frac{v^2\sin^2 2\beta}{v_\phi^2+v^2s_{2\beta}^2}(p+q)_\mu(p+q)_\nu$
H	G_+	$-gW_+^\alpha H \partial_\alpha G_-$	$-g^2(p+q)_\mu(p+q)_\nu$
G_+	G_0	$\frac{i}{2}gW_+^\alpha G_0 \overset{\leftrightarrow}{\partial}_\alpha G_-$	$\frac{g^2}{4}(2p+q)_\mu(2p+q)_\nu$